**CHAPTER 5**

**CAPITAL BUDGETING / INVESTMENT APPRAISAL  UNDER CERTAINTY**
**(FUNDAMENTAL)**

This topic is one that can confidently be referred to as the foundation or backbone of financial management around the world, and the reason for this is not farfetched. Financial management deliberates on the judicious use of the organisation's funds on a worthwhile or profitable investment. The question therefore is, how can you possibly determine a profitable investment without first appraising  it? I am sure you are getting my drift of the importance of this topic.

In  this  chapter, I only wish to lay a solid foundation on different concepts of capital budgeting with specific emphasis on ensuring that rookie or newbie gets the core knowledge needed to ace this course. Then when the foundation has been well laid, we shall now progress to the advance stage of capital budgeting. By advance, I don't mean 'advanced advanced", since it will be way beyond the scope of this book and may end up jeopardizing the purpose of simplicity. So, the fundamental stage is for a fresher in the university, while the advanced stage is for MBA students, got that?. Alright, lets ride!

## 5.1 CONCEPT OF CAPITAL BUDGETING

Most times, organisations are engaged in different investment activities that revolve around replacement of machinery, development and innovation of new products, establishment of new plants or research and development. These projects are usually funded by the organisation's long term funds (debt, equity or even retained earnings), since these projects usually have a life span that exceeds one year. So, when an organisation undertakes a planning process that involves determining whether it should invest in any of these projects [as mentioned above], by appraising the feasibility of such project(s) on the basis of the ability of those investments to increase the net worth of that organisation or to increase the wealth of shareholders, that organisation is said to be engaged in capital budgeting or investment appraisal.
Capital budgeting may be defined as the investment of financial resources on a project with the aim of realizing benefits (profits) in the future over a period of time. It may also be defined as the process of conceiving, analyzing, evaluating and selecting the most profitable project for investment.
Ever wonder why it is called capital budgeting under certainty? Well, it is so called because it is assumed that the investor who is budgeting for capital has a foreknowledge of the market conditions, is aware of investment opportunities, or can even determine or accurately predict expected returns from each of the investment.

## 5.2 INVESTMENT DECISION PROCESS

The process of investment decision are:
1. Identify potential or possible investment projects.
2. Identify alternatives to the projects.
3. Source for data /information on projects.
4. Evaluate data based on available projects.
5. Select the best project.
6. Implement the project.
7. Project monitoring and control.

## 5.3 TYPES OF INVESTMENT DECISIONS

These are varying types of investment decisions, the most common being:
1. Expansion of existing business.

2. Creation of new business.
3. Replacement and modernization.
4. Automating a process.
5. Market development and new product introduction.

## 5.4 CHARACTERISTICS OF A GOOD CAPITAL BUDGETING DECISION
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1. They involve a commitment of a huge amount of funds.
2. They are mostly irreversible.
3. It should state projects to be accepted or rejected.
4. It can be applied to all capital budgeting decisions.
5. It should recognize the time value of money.
6. It should help in the ranking of projects.
7. The project must realize future benefits over a reasonable long period into the future.
8. It should aid the decision maker in choosing the best project from competing projects.

## 5.5    PROJECTS CLASSIFICATION

1. **Independent projects:** projects are said to be independent if the acceptance or rejection of one project does not affect the acceptance or rejection of the other. Let me explain further, it means if I decide to accept project A, it does not disturb me from accepting project B, especially if both projects A and B have positive NPVs or if I have the capital to execute both projects.  It means if I accept project A, I can also accept project B, on the condition that they are not competing projects, if projects A and B is positive and if I have the financial muscle to muster both projects.

2. **Mutually exclusive projects:** unlike independent project, mutually exclusive projects constrain or confine someone to a choice. Projects are said to be mutually exclusive if the acceptance of one project only leads us to the rejection of the other. It means even if we have enough capital to accept both projects and they both have positive NPVs, it is one and only one project that can be accepted, that is, the better project. In that case, the other must be rejected. Let me paint a clearer picture. Imagine two cities A and B that intends to start trading with each other but are divided by a lagoon. They have the choice of either building a bridge across the lagoon or making a boat to aid transportation of their goods, but not to do both. Inadvertently, both cities are involved in mutually exclusive projects.

However, there is one important information I want you to take home, that is if you are not reading this book at home already (laughs).  Please note that when you hear of the conflict exiting between NPV and IRR, always know that the root cause of that conflict as regards which is better,  is due to the fact that both projects are mutually exclusive. If they were independent projects there wouldn't be a need for conflict, since both project can be selected. I will discuss extensively on this much later in the "advance capital budgeting topic" subsequently.

3. **Conventional or Normal cash flow projects:** Put succinctly, these are projects that initial have outlays or outflows, and are then followed by inflows in the successive years without any interruption of outflows. They have a high degree of certainty or generating positive cash flows over a short period of time. e.g.

| Period | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| Cash flows | -1,000 | 200 | 500 | 900 | 300 | 750 |

Or

| Period | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| Cash flows | -1,000 | -200 | 500 | 900 | 300 | 750 |

4. **Non-Conventional or Non-Normal cash flow projects:** These are projects that have outflows even after inflows have begun. It is an admixture of both inflows and outflows in no particular order e.g.
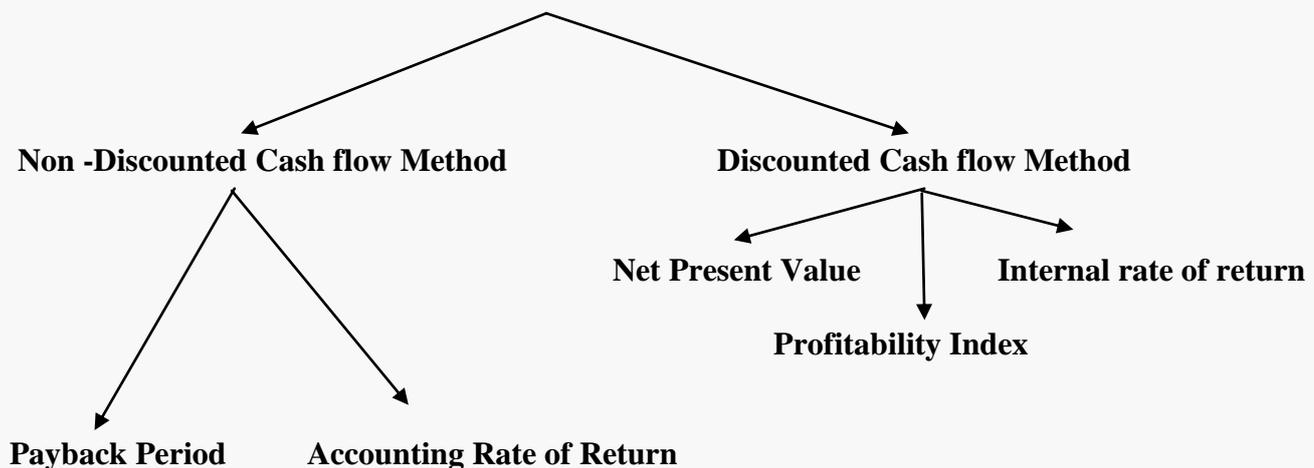
| Period | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| Cash flows | -1,000 | 200 | 500 | 900 | 300 | -750 |

Or

| Period | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| Cash flows | -1,000 | -200 | 500 | -900 | 300 | 750 |

Just like the mutually exclusive project, they also have their own down sides. On a good day, a project can only have one NPV and one IRR. But for the non-conventional or non-normal projects the reverse seems to be the case. Non-conventional projects have the tendency of providing 'multiple IRRs' for one project". See advance capital budgeting topic for example and explanation.

### TECHNIQUES FOR EVALUATING PROJECTS

**Non -Discounted Cash flow Method**

**Discounted Cash flow Method**

**Net Present Value**

**Internal rate of return**

**Profitability Index**

**Payback Period**      **Accounting Rate of Return**

**Note:** The Payback period and the Accounting rate of return are called the traditional method.

## 5.6 PAYBACK PERIOD

It is defined as the time required by an investor or an organization to recover the total capital outlay invested from the stream cash flows. This technique seeks to determine the period (in years or the number of years), it will take a project to recover or repay the initial cost or outlay invested in that

project. It should be noted that if a project has a fixed cash inflows , then the payback period will be expressed as:

=       Project cost or outlay
         Annual Cash inflows

## 5.7 ADVANTAGES OF P.B.P

1.  It is used when the degree of risk is high.
2.  It shows how soon it will take to recover the cost of purchasing an asset.
3.  It is simple to use and understand.
4.  It acts as a yardstick for comparing the profitability of two projects.

## 5.8 DISADVANTAGES OF THE P.B.P

1.  It does not consider the time value of money.
2.  It does not state if the wealth of the shareholders is being maximized.
3.  It does not consider the total cash flows of the project.
4.  It does not take into account the cash flows that may arise from the scrap value at the end of the project's life, particularly if the scrap value is very significant.
5.  It fails to realize that actual profitability is based on the number of years it will keep operating after the payback period.

## ILLUSTRATION 1

Mr. John Smith is considering an investment project costing $100,000. The cash inflows for the project are estimated at $25,000 annually for 7 years. How long will it take for the  initial outlay to be repaid if the payback period (P.B.P) is used to appraise the project?

## SOLUTION

**How to solve**

**METHOD 1**
P.B.P =   Project cost or outlay
             Annual Cash inflows

        =       $100,000
                 $25, 000

Payback period =   4 years.

Note: The 7 years given in the question is immaterial in this case, since there is a constant cash flow.

**DECISION**
Mr. John Smith should accept the project since he will be able to recover the initial outlay in 4 years time.

Note: I must warn you that this formula can only be applied or used when the annual cash flows are the same or constant all through. Let's check out this other method.

**METHOD 2**
Draw four columns in the following order:
Column 1 = Period
Column 2 = Cash Inflows
Column 3 = Payment.
Column 4 = Balance/ Year Ended.

The first step is to input the number of years in the period column starting from 0. The zero represents the time or beginning when the outlay or cost of the project was taken out of the pocket or account and set aside for the purpose of investment.

| Period/year | Cash inflows $ | Payment $ | Bal/year ended $ |
|---|---|---|---|
| 0 | | | |
| 1 | | | |
| 2 | | | |
| 3 | | | |
| 4 | | | |
| 5 | | | |
| 6 | | | |
| 7 | | | |

The next step is to input the cash inflows in the second column as provided in the question. i.e. 25,000 each year for 7 years, now the 7 years becomes relatively material or important in this context.

| Period/year | Cash inflows $ | Payment $ | Balance/year ended $ |
|---|---|---|---|
| 0 | - | | (100,000) |
| 1 | 25,000 | | |
| 2 | 25,000 | | |
| 3 | 25,000 | | |
| 4 | 25,000 | | |
| 5 | 25,000 | | |
| 6 | 25,000 | | |
| 7 | 25,000 | | |

You will notice that in period 0, in the cash inflows column what you have there is a dash meaning that there was no cash inflow but outflows, which has to be paid for. This goes into the balance column and will be bracketed indicating cost.

The next step is to start "paying back" the outlay with each year's respective cash inflows, then writing its balance in the 4$^{th}$ or balance column. See below for more explanation.

For year 1 below, he paid $25,000 from a cost of $100,000 the balance is $75,000. See below:

| Period/year | Cash inflows $ | Payment $ | Bal/year ended $ |
|---|---|---|---|
| 0 | - | - | (100,000) |
| 1 | 25,000 | 25,000 | (75,000) |
| 2 | 25,000 | | |
| 3 | 25,000 | | |
| 4 | 25,000 | | |
| 5 | 25,000 | | |
| 6 | 25,000 | | |
| 7 | 25,000 | | |

For year 2 below, he paid $25,000 again from a cost of $75,000 the balance is $50,000. See below:

| Period/year | Cash inflows $ | Payment $ | Balance/year ended $ |
|---|---|---|---|
| 0 | - | - | (100,000) |
| 1 | 25,000 | 25,000 | (75000) |
| 2 | 25,000 | 25,000 | (50,000) |
| 3 | 25,000 | | |
| 4 | 25,000 | | |
| 5 | 25,000 | | |
| 6 | 25,000 | | |
| 7 | 25,000 | | |

For year 3 below, he paid $25,000 again from a cost of $50,000 the balance is $25,000. See below:

| Period/year | Cash inflows $ | Payment $ | Balance/year ended $ |
|---|---|---|---|
| 0 | - | - | (100,000) |
| 1 | 25,000 | 25,000 | (75,000) |
| 2 | 25,000 | 25,000 | (50,000) |
| 3 | 25,000 | 25,000 | (25,000) |
| 4 | 25,000 | | |
| 5 | 25,000 | | |
| 6 | 25,000 | | |
| 7 | 25,000 | | |

For year 4 below, he paid $25, 000 again from a cost of $25, 000 the balance left is now is 0. Meaning he paid back the initial cost outlay of $100, 000 after 4 years. The remaining cash flows therefore become his. So, since he can pay back his initial outlay, the project should be accepted.

| Period/year | Cash inflows $ | Payment $ | Balance/year ended $ |
|---|---|---|---|
| 0 | - | - | (100,000) |
| 1 | 25,000 | 25,000 | (75000) |
| 2 | 25,000 | 25,000 | (50,000) |
| 3 | 25,000 | 25,000 | (25,000) |
| 4 | 25,000 | 25,000 | - |
| 5 | 25,000 | | |
| 6 | 25,000 | | |
| 7 | 25,000 | | |

## ILLUSTRATION 2

James Brown invested in a new business machinery costing $500,000. The annual cash flows of the machinery are as follows:

| Year 1 | $100,000 |
|---|---|
| Year 2 | $150,000 |
| Year 3 | $80,000 |
| Year 4 | $80,000 |
| Year 5 | $90,000 |
| Year 6 | $85,000 |

Determine the viability of the project using payback period for the project.

## SOLUTION
**How to solve**
The previous processes followed will be used in solving this question.

| Period/year | Cash inflows $ | Payment $ | Balance/year ended $ |
|---|---|---|---|
| 0 | - | - | (500,000) |
| 1 | 100,000 | 100,000 | (400,000) |
| 2 | 150,000 | 150,000 | (250,000) |
| 3 | 80,000 | 80,000 | (170,000) |
| 4 | 80,000 | 80,000 | (90,000) |
| 5 | 90,000 | 90,000 | - |
| 6 | 85,000 | | |

The P.B.P = 5 years. The years with the dash (-).

Decision: Accept project because the initial outlay can be paid back in 5 years time.

**EXPLANATION**

The total outlay for the project was $500,000 in the balance column. If he paid $100,000 that came in as the cash inflow for the 1$^{st}$ year, the balance now becomes $400,000. i.e. The amount owing or left to be paid. Furthermore, $150,000 was paid from the balance of $400,000 in the first year, leaving him with a balance of $250,000. In the same vein, $80,000 was paid in the 3$^{rd}$ year from a previous balance of $250,000 that gave a new balance of $170,000. Again, he paid the second $80,000 in the 4$^{th}$ year from the previous balance of $170,000 to leave him with a balance of $90,000. The final balance of $90,000 was paid with the cash inflow of the 5$^{th}$ year, leaving with no balance. It therefore means that it took James Brown 5 years to payback the initial outlay. Therefore, James Brown is advised to accept the project on the basis of payback period.

**ILLUSTRATION 3**

Janet Pierce was advised by her brother to invest in a certain lucrative project gulping $50,000. The annual returns on investment are given as follows:

| YEAR | RETURN ($) |
|------|------------|
| 1 | 30,000 |
| 2 | 15,000 |
| 3 | 12,000 |
| 4 | 8,000 |

Use the payback period to determine the feasibility of the project.

**SOLUTION**

**How to solve**

| Period/year | Cash inflows $ | Payment $ | Balance/year ended $ |
|-------------|----------------|-----------|----------------------|
| 0 | - | - | (50,000) |
| 1 | 30,000 | 30,000 | (20,000) |
| 2 | 15,000 | 15,000 | (5,000) |
| 3 | 12,000 | | |
| 4 | 8,000 | | |

If you take a very critical look at the balance of the 2$^{nd}$ year, you will notice that it's $5000. However, funny enough, the cash inflow for the subsequent year is $12,000, quite confusing you might say i.e. the amount or balance she has left to pay is less than the cash inflows to be used for it's payment.

Don't worry, you will learn how this is done right away. Anytime you see this type of situation, all you need do is to put the balance to be paid as numerator and the cash inflows to be used for payment in that year as the denominator, in the payment column for that year, as shown below:

| Period/year | Cash inflows $ | Payment $ | Balance/year ended $ |
|---|---|---|---|
| 0 | - | - | (50,000) |
| 1 | 30,000 | 30,000 | (20,000) |
| 2 | 15,000 | 15,000 | (5,000) |
| 3 | 12,000 | $\frac{5,000}{12,000} = 0.42$ | - |
| 4 | 8,000 | | |

Therefore, the payback period is 2 years + 0.42 years = 2.42 years. Some authors/lecturers will prefer that you express your answers in years and months. To achieve this, all you need to do is to multiply the last calculation by 12 months. See below:

| Period/year | Cash inflows $ | Payment $ | Balance/year ended $ |
|---|---|---|---|
| 0 | - | - | (50,000) |
| 1 | 30,000 | 30,000 | (20,000) |
| 2 | 15,000 | 15,000 | (5,000) |
| 3 | 12,000 | $\frac{5,000}{12,000}$ x 12 = 5 | - |
| 4 | 8,000 | | |

P.B.P = 2 years + 5 months

Therefore, the payback period is approximately 2 years 5 months. The question now is should Janet Pierce accept the or reject the project? Janet Pierce will accept the project since she can pay back the capital outlay.

**ILLUSTRATION 4**

Kenneth Kuffor is faced with a major executive decision about the choice of a project from two competing projects costing $120,000 each. The criteria for the choice of a better project are based on the payback period. The following data are related to these individual projects that are MUTUALLY EXCLUSIVE.

| | PROJECT A $ | PROJECT B $ |
|---|---|---|
| Yr1 | 40,000 | 100,000 |
| Yr2 | 50,000 | 80,000 |
| Yr3 | 60,000 | 40,000 |
| Yr4 | 70,000 | 30,000 |

You are required to determine a better profit using P.B.P.

**SOLUTION**
**How to solve**

I guess you now know the meaning of mutually exclusive, from the explanation made earlier. I must also state here that since there are two projects (A and B) they will be calculated separately.
Let's start our calculations.

| Project A | | | | Project B | | | |
|---|---|---|---|---|---|---|---|
| Period | Cash flows $ | Payment $ | Balance $ | Period | Cash flow $ | Payment $ | Balance $ |
| 0 | - | - | (120,000) | 0 | - | - | (120,000) |
| 1 | 40,000 | 40,000 | (80,000) | 1 | 100,000 | 100,000 | (20,000) |
| 2 | 50,000 | 50,000 | (30,000) | 2 | 80,000 | 20,000 80,000 = 0.25 | |
| 3 | 60,000 | 30,000 60,000 =0.5 | - | 3 | 40,000 | | |
| 4 | 70,000 | | | 4 | 30,000 | | |

A's P.B.P is 2 + 0.5 = 2.5 years or 2 years 6 months

B's P.B.P is 1 + 0.25 = 1.25 years or 1 year 3 months

From the above we can deduce that both projects can payback their outlays, but the better of the two must be selected, since they are mutually exclusive.
So what do you think Mr. Kenneth Kuffor should do? He should accept Project B, and this is based on the fact that it has an earlier or shorter payback period than project A.

**5.9   USING THE CUMULATIVE METHOD FOR PAYBACK PERIOD**
Another popular method that can be used for solving payback period is using the cumulative method.

**ILLUSTRATION 5**

Using a previously solved example, James Brown invested in a new business machinery costing $500, 000. The annual cash flows of the machinery are as follows:

| Period | Cash inflows |
|---|---|
| Year 1 | $100,000 |
| Year 2 | $150,000 |
| Year 3 | $80,000 |
| Year 4 | $80,000 |
| Year 5 | $90,000 |
| Year 6 | $85,000 |

Determine the payback period of the project.

**SOLUTION**
**How to solve**
Draw three columns in the following order:
Column 1 = Period
Column 2 = Cash Inflows
Column 3 = Cumulated cash flows

The first step is to input the number of years in the period column starting from 1.

| Period/year | Cash inflows $ | Cumulated Cash flows $ |
|---|---|---|
| 1 | | |
| 2 | | |
| 3 | | |
| 4 | | |
| 5 | | |
| 6 | | |

The second step is to input the cash inflows in the second column as provided in the question. See table below:

| Period/year | Cash inflows $ | Cumulated Cash flows $ |
|---|---|---|
| 1 | 100,000 | |
| 2 | 150,000 | |
| 3 | 80,000 | |
| 4 | 80,000 | |
| 5 | 90,000 | |
| 6 | 85,000 | |

The third step is to cumulate the cash flows:

| Period/year | Cash inflows $ | Cumulated Cash flows $ |
|---|---|---|
| 1 | $100,000 | $100,000 |
| 2 | $150,000 | $250,000 |
| 3 | $80,000 | $330,000 |
| 4 | $80,000 | $410,000 |
| 5 | $90,000 | $500,000 |
| 6 | $85,000 | $585,000 |

The fourth step, look for the period where the cumulated value is equal to the initial outlay of $500,000 to know the payback period.

In this example, the payback period is 5 years .

You can use any method approved in your school, but the first method is commonly used.

## ILLUSTRATION 6

Kenneth Kuffor is faced with a major executive decision about the choice of a project from two competing projects costing $120,000 each. The criteria for the choice of a better project are based on the payback period. The following data are related to these individual projects that are MUTUALLY EXCLUSIVE.

| | PROJECT A $ | PROJECT B $ |
|---|---|---|
| Yr 1 | 40,000 | 100,000 |
| Yr 2 | 50,000 | 80,000 |
| Yr 3 | 60,000 | 40,000 |
| Yr 4 | 70,000 | 30,000 |

Recommend the better project using the Payback period.

## SOLUTION

**How to solve**
**Step 1**: Cumulate the individual projects' cash flows

| Period/year | Cash inflows A $ | Cumulated A $ | Cash inflows B $ | Cumulated B $ |
|---|---|---|---|---|
| 1 | 40,000 | 40,000 | 100,000 | 100,000 |
| 2 | 50,000 | 90,000 | 80,000 | 180,000 |
| 3 | 60,000 | 150,000 | 40,000 | 220,000 |
| 4 | 70,000 | 220,000 | 30,000 | 250,000 |

Step 2: Check the cumulative columns of project A and B for any one that has the outlay of $120,000

NOTE: If you check the cumulated cash flows columns for A and B, you will notice that there is no point where any value is exactly equal to the initial outlay of $120,000.

So we compute accordingly. Solving for Project A.
1. Look at the cumulated column of A for the value that is closest to $120,000 i.e. the initial outlay. Note that the value has to be lesser than the initial outlay. For cumulative column of project A the value that is closer to the initial outlay is $90,000, occurring in period 2.

2. Deduct this $90,000 from the initial outlay. ($120,000 – $90,000) = $30,000.

| Period/year | Cash inflows A $ | Cumulated A $ | |
|---|---|---|---|
| 1 | 40,000 | 40,000 | |
| 2*** | 50,000 | 90,000 | 120,000 – 90,000 = 30,000 |
| 3 | 60,000 | 150,000 | |
| 4 | 70,000 | 220,000 | |

3. Now make this $30,000 the numerator of the next immediate cash flow in "Cash inflows A" column i.e. $60,000 multiplied by 12 months. See below:
= $\frac{\$30,000}{\$60,000}$ x 12 months = 6 months

4. The payback period is the period where the closest value to the initial outlay occurred i.e. period 2 , the one asterisk above + the months calculated.

The payback period for Project A = 2 years 6 months.

Solving for Project B
1. Look at the cumulated column of project B for the value that is the closest to $120,000 i.e. the initial outlay. Note also that the value has to be lesser than the initial outlay. For cumulative B, the value is $100,000, occurring in period 1.

2. Deduct this $100,000 from the initial outlay. ($120,000 – $100,000) = $20,000

| Period/year | Cash inflows B $ | Cumulated B $ | |
|---|---|---|---|
| 1*** | 100,000 | 100,000 | 120,000 – 100,000 = 20,000 |
| 2 | 80,000 | 180,000 | |
| 3 | 40,000 | 220,000 | |
| 4 | 30,000 | 250,000 | |

3. Now make this $20,000 the numerator of the next immediate cash flow in the "Cash inflows B" column i.e. $80,000 multiplied by 12 months.
= $\frac{\$20,000}{\$80,000}$ x 12 months = 3 months

4. The payback period is the period where the closest value to the initial outlay occurred i.e. period 1 asterisked above + the months calculated.

The payback period for Project B = 1 year 3months

Now comparing both projects (A and B's) payback periods, the better project is the one with the lesser payback period i.e. Project B.

## ILLUSTRATION 7
Professor Bush is planning to invest in a project costing $75,000. The life span of the project is 4 years, and has the following information:

| Year | Cash flow $ |
|---|---|
| 1 | 35,000 |
| 2 | 15,000 |
| 3 | 10,000 |
| 4 | 10,000 |

Using the payback period determine the viability of the project.

**SOLUTION**

| Period/year | Cash inflows $ | Payment $ | Balance/year ended $ |
|---|---|---|---|
| 0 | - | - | (75,000) |
| 1 | 30,000 | (35,000) | (40,000) |
| 2 | 15,000 | (15,000) | (25,000) |
| 3 | 10,000 | (10,000) | (15,000) |
| 4 | 10,000 | (10,000) | (5,000) |

Notice anything funny about this solution? At the end of the 4$^{th}$ year, we still have a balance of $5, 000 yet unpaid. In this case, all we need do is to reject the project since the outlay cannot be paid, "we cannot recoup our investment".

Note: there is a concept know as the **discounted payback period**. Please check that out under the topic "capital budgeting under uncertainty" in the later part of this book.

**5.10 DISCOUNTED CASH FLOW METHOD**

It is a capital budgeting technique which considers the time value of money and the total benefit accrued to that project during the period of its existence. Now, let's chit-chat a bit about discounted cash flow. Since you are just learning this I need to explain it as much as possible. The cash flows provided for the purpose of appraising project under this technique are usually the **after-tax cash flows.** This means, we are not using profit or cash flows that include tax. Furthermore, the cash flows are future value cash flows or cash flows at the end of each period, that is why it is certainty in the first place.
 Since, cash flows provided are futuristic, we therefore need to discount these cash flows to the present values or "the now, this instant, as in this moment" , bringing the future to the now. So when we use the cost of capital to discount or "reduce" the future value to the present value, then we have discounted the cash flows.
I am sure at some point in your class, you would have heard something like " should I give you $100 now or give you the same $100 in 12 months time, puzzled right? Well, it is better you take that $100 now, because you can invest that money in an account that can grow it to $109 at the end of the 12 months period. Asides that, uncertainty of the future may render the future $100 worthless. What if inflation rate rises astronomically, it obviously affects the $100 negatively by reducing its actual value. On the other hand, what if the person promising you the $100, dies on or before the agreed date, you lose all your stakes. So, can you now see the importance of time value of money? The major discounted cash flow methods are:

- 1. NPV = Net Present Value.
- 2. IRR = Internal Rate of Return.
- 3. PI   = Profitability Index.

**5.11 NET PRESENT VALUE**
The Net Present Value or the NPV as is popularly called is an investment appraisal technique used to assess the worth, viability or profitability of a project by finding the difference between the present value of cash outflows and the present value of cash inflows over a period of time. The edge this technique has over other techniques is based on the fact that it takes the **time value of money** into consideration, it also considers all the after-tax cash flows and not profit. This technique is the best for

capital investment appraisal. It has been argued that whenever there is a conflict or rivalry amongst the techniques in terms of superiority, then the NPV will be selected **because it has the characteristics of adding value. Most importantly, note that the NPV is an absolute investment appraisal technique while the IRR is a relative appraisal technique.**

Decision rule:
If calculated NPV > 0 = Accept the project.
If calculated NPV< 0  = Reject the project.
If calculated NPV = 0, then it is no more NPV but IRR.

## 5.12 ADVANTAGES OF THE NET PRESENT VALUE

1   It takes the time value of money into account.
2   It tells if the shareholders' wealth is maximized or not.
3   It considers all the after-tax cash flows of the whole project.
4   It aid or eases the comparison of two projects.

## 5.13 DISADVANTAGES OF THE NET PRESENT VALUE.

1   It assumes that the discount rate is known and constant.
2   Its result cannot be termed the best when there are more than one projects with different outlays.
3   It does not render an accurate picture in case of alternative projects or where projects with limited funds have unequal lives.
4   It is prone to forecasting error.

## ILLUSTRATION 1

Kunle Amusan is considering a project whose capital outlay is $20,000. If the cash flows to be generated for the next 4 years are $10,000, $20,000, $15,000, and $8,000 respectively. Using a 10% cost of capital, calculate the Net Present Value (NPV) of the project suggesting whether the project be accepted or rejected.

## SOLUTION

**How to solve**

$$NPV = \sum \frac{A_t}{(1+r)^t} - Co$$

**Where   $A_t$ = cash flow per period**
**$t$ = period or year**
**$r$ = cost of capital or discount rate**
**$Co$ = capital outlay**

**Method 1**

1.  Write down the cash flows horizontally with a (+) sign in between them minus the initial outlay. See below:

    Yr 1      yr2      yr3      yr4

    (10,000 + 20, 000 + 15,000 + 8,000) - 20,000

2.  Divide each of the cash flows by the discounting factor.
    $(1 + r)^t = (1 + 10\%) = (1 + 0.10) = (1.1)$

    Yr 1      yr2      yr3      yr4

    $$\frac{10,000}{(1.1)} + \frac{20,000}{(1.1)} + \frac{15,000}{(1.1)} + \frac{8,000}{(1.1)} - 20,000$$

Note the '1' that comes before the 10% in point 2 above is always constant and the 'r' represents the cost of capital given in the question.

3.  Now the next step is to raise the denominators or discounting factors to the power of their respective years as shown below:

    Yr 1      yr2      yr3      yr4

    $$\frac{10,000}{(1.1)^1} + \frac{20,000}{(1.1)^2} + \frac{15,000}{(1.1)^3} + \frac{8,000}{(1.1)^4} - 20,000$$

4.  Then we use scientific calculator to get the value of the project. If you divide correctly you should have.

    Yr 1      Yr2      Yr3      Yr4

    $9090.91 + 16,528.93 + 11,269.72 + 5464.11 - 20,000$

    $NPV = \$42,353.67 - \$20,000 = \$22,353.67$

**Decision**: Accept the project since the NPV is positive and it increases Kunle Amusan's wealth by **$22,353.67**.

Due to the "over simplicity" of this method, it has severally been discouraged by lecturers in many higher institutions, therefore giving rise to the second method below. You can however use it, if the method is accepted in your university, polytechnic or college of education.

**Method 2**
Draw four columns in the following order:
Column 1 = Period.
Column 2 = Cash flows.
Column 3 = Discounting factor.
Column 4 = Present value.

The first step is to input the number of years in the period column starting from 0.   Period zero represents the time or the beginning when the capital outlay was taken out of the pocket or account and set aside for investment.

| Period | Cash flows $ | Discounting factor | Present value $ |
|--------|--------------|--------------------|-----------------|
| 0 |  |  |  |
| 1 |  |  |  |
| 2 |  |  |  |
| 3 |  |  |  |
| 4 |  |  |  |

The second step is to input the cash inflows in the second column as provided in the question.

| Period | Cash flows $ | Discounting factor 10% | Present value $ |
|--------|--------------|------------------------|-----------------|
| 0 | (20,000) |  |  |
| 1 | 10,000 |  |  |
| 2 | 20,000 |  |  |
| 3 | 15,000 |  |  |
| 4 | 8,000 |  |  |

Note that the cash outlay or initial outlay as it were, will be written in the second column beside period zero and bracketed. See table above.

The third step is to calculate the discounting factor and input same in the third column. Nevertheless, let me give you a quick rundown of how to calculate the discounting factor:

The formula is $\dfrac{1}{(1+r)^n}$  or  $(1+r)^{-n}$

Where 1 is always constant
$r = 10\% = (10/100) = 0.1$ (as given in the question)
$n$ = the period or years you are calculating for. See below:

Using $(1+r)^{-n}$  to calculate for 10% discounting factor.

Period 0 $= (1+0.1)^{-0} = 1$
Period 1 $= (1+0.1)^{-1} = 0.9091$
Period 2 $= (1+0.1)^{-2} = 0.8264$
Period 3 $= (1+0.1)^{-3} = 0.7513$
Period 4 $= (1+0.1)^{-4} = 0.6830$

Note: I stopped at period 4, because the number of  periods provided in the question were 4 , if they were more, I would have continued.
Now, I will like to let you in on a secret of calculating the discounting factor using **the jet age method**. Imagine that you have a cash flow of say 40 years for which you must calculate their discounting

factors; does it mean that you have to go through all that rigor like the one in the illustration above? No, you shouldn't. So let's see a faster way of doing this.

I am sure you know that the discounting value for this question is (1+ 0.1) which is the same as (1.1), agreed? O.k. using your scientific calculator do the following:

1. Let's first start solving for period 0, press 1 divided by (1.1) raise to power 0 i.e. $1/ (1.1)^0$ you will get '1' as your answer.
2. For period 1, with that '1' still appearing on your calculator, don't clear it on the screen of the calculator divide it by 1.1 again and get 0.909090 or 0.9091.
3. For period 2 press "=" sign you will get 0.8264. Then keep pressing the "(=)" sign one after the other for each period until you get to the required number of years. This is the simplest and fastest way to solve for discounting factor. Keep trying it. If you like this book why not talk to your friends or group about it.

Let's continue our solution.

These values will now be inputted in the discounting factor column. See table below:

| Period | Cash flows $ | Discounting factor 10% | Present value $ |
|---|---|---|---|
| 0 | (20,000) | $(1+ 0.1)^{-0} = 1$ | |
| 1 | 10,000 | $(1+ 0.1)^{-1}= 0.9091$ | |
| 2 | 20,000 | $(1+ 0.1)^{-2}=0.8264$ | |
| 3 | 15,000 | $(1+ 0.1)^{-3}=0.7513$ | |
| 4 | 8,000 | $(1+ 0.1)^{-4}=0.683$ | |

The next step is to multiply **each cash flow** by their respective **discounting factors** then sum them up so that you can get the **Net Present Value.** See table below.

| Period | Cash flows $ | Discounting factor 10% | Present value $ |
|---|---|---|---|
| 0 | (20,000) | 1 | (20, 000) |
| 1 | 10, 000 | 0.9091 | 9091 |
| 2 | 20, 000 | 0.8264 | 16, 528 |
| 3 | 15, 000 | 0.7513 | 11, 269.5 |
| 4 | 8, 000 | 0.6830 | 5464 |
| **NPV** | | | **22, 352.5** |

To get the answer using your calculator, note that the ($20,000) takes on a negative sign so you press the values in this order on your calculator as shown below:

-20, 000 + 9091 + 16528 + 11269.5 + 5464 = 22, 352.5

The first method i.e. the non-tabular method. The slight difference is occasioned by rounding off the discounting factor, which is quite acceptable.

**Decision:** Accept the project in as much as the NPV is positive and that it increases Kunle Amusan wealth by $22,352.50.

## ILLUSTRATION 2

Dennis Henchman is an investor in the quarry industry. He plans to invest $80,000 on an equipment that has 5 years life span and a residual or salvage value of $5,800. If the cash inflows of the project are as follows:

| Year | Cash flow $ |
|------|-------------|
| 1 | 10,000 |
| 2 | 20,000 |
| 3 | 20,000 |
| 4 | 10,000 |
| 5 | 10,000 |

Use the Net Present Value to determine the acceptability of the project at a 15% cost of capital.

## SOLUTION

**How to solve**

$$NPV = \sum \frac{A_t}{(1+r)^t} - Co$$

Where  $A_t$ = cash flow per period
 $t$ = period or year
 $r$ = cost of capital or discount rate
 $Co$ = capital outlay

| Period | Cash flows $ | Discounting factor @ 15% | Present value $ |
|--------|--------------|--------------------------|-----------------|
| 0 | (80,000) | $(1+0.15)^{-0} = 1$ | (80,000) |
| 1 | 10,000 | $(1+0.15)^{-1} = 0.8696$ | 8,696 |
| 2 | 20,000 | $(1+0.15)^{-2} = 0.7561$ | 15,122 |
| 3 | 20,000 | $(1+0.15)^{-3} = 0.6575$ | 13,150 |
| 4 | 10,000 | $(1+0.15)^{-4} = 0.5718$ | 5,718 |
| 5 | 10,000 + 5,800 = 15,800 | $(1+0.15)^{-5} = 0.4972$ | 7,855.76 |
| NP | | | (29,458.24) |

**Decision:** Dennis Henchman should reject the project because the net present value is negative and that it will decrease his wealth by $29,458.24

There are some significant points I will like to make here:

1. Note that in the 5<sup>th</sup> period something happened. What happened was that after using up the machine, Dennis decided to sell it off as scrap. It means that the annual cash inflow for the

5th year will be $10,000 + the salvage value $5,800, which gave a new cash inflow of $15,800 that was used.

2. The salvage value can also be called, **resale value, scrap value or residual value.**
3. The salvage value is most time added at the end of the period given in the question, **except otherwise stated.**

## ILLUSTRATION 3

Mr. Kenneth Kuffor still faced with the challenge of selecting a project from two MUTUALLY EXCLUSIVE projects that both have 4 years life span each. They both have identical outlay of $120,000 each at a cost of capital of 10% for project A and 15% for project B. Which of the projects should be selected using the NPV criteria, given the information below?

| Year | Project A cash flow $ | Project B cash flow $ |
|------|-----------------------|-----------------------|
| 1 | 40,000 | 100,000 |
| 2 | 50,000 | 80,000 |
| 3 | 60,000 | 40,000 |
| 4 | 70,000 | 30,000 |

## SOLUTION

**How to solve**

$$NPV = \sum \frac{A_t}{(1+r)^t} - Co$$

Where $A_t$ = cash flow per period
$t$ = period or year
$r$ = cost of capital or discount rate
$Co$ = capital outlay

### Project A

| Period | Cash flows $ | D. c. f 10% | PV $ |
|--------|--------------|-------------|------|
| 0 | (120,000) | 1 | (120,000) |
| 1 | 40,000 | 0.9091 | 36364 |
| 2 | 50,000 | 0.8264 | 41320 |
| 3 | 60,000 | 0.7513 | 45078 |
| 4 | 70,000 | 0.6830 | 47810 |
| NPVs | | | $50,572 |

Project B

| Period | Cash flows $ | D.c.f 15% | PV $ |
|--------|--------------|-----------|------|
| 0 | (120,000) | 1 | (120,000) |
| 1 | 100,000 | 0.8696 | 86,960 |
| 2 | 80,000 | 0.7561 | 60,488 |
| 3 | 40,000 | 0.6575 | 26,300 |
| 4 | 30,000 | 0.5718 | 17,154 |
| | | | $70,902 |

**Decision:** Mr. Kenneth should select project B because it gives a higher NPV when compared with A's NPV.

**Note: If the project were independent projects, the investor can choose both project since they both have positive NPVs.**

## ILLUSTRATION 4

Mr. Ariel is planning to invest in a project valued at $150,000. The life cycle of the project is 4 years and has the following information:

| Year | Revenue $ | Incurred expenses $ |
|------|-----------|---------------------|
| 1 | 145,000 | 45,000 |
| 2 | 120,000 | 40,000 |
| 3 | 100,000 | 60,000 |
| 4 | 70,000 | 40,000 |

Use the NPV to determine the acceptability of the project at a discount rate of 15 %.

## SOLUTION

**How to solve**

$$NPV = \sum \frac{A_t}{(1+r)^t} - Co$$

Where
$A_t$ = cash flow per period
$t$ = period or year
$r$ = cost of capital or discount rate
$Co$ = capital outlay

This illustration is a little bit technical. This is based on the fact that most students would worry as to which information to make use of, or wonder what manipulations are to be made to get the right answer to this question.

It's quite easy, I would want you to follow these procedures, and note the adjustment made to the conventional table.

Note: You have to deduct the each incurred expense from its respective revenue so that you get values to be written in the net cash flows column, which is then used for multiplying the discounting factor to get the present value. See table below:

| Period | Revenue $ | Incurred expenses $ | Net cash flows $ | Discounting factor 15% | Present values $ |
|--------|-----------|---------------------|------------------|------------------------|------------------|
| 0 | - | - | (150,000) | 1 | (150,000) |
| 1 | 145,000 | (45,000) | 100,000 | 0.8696 | 86,960 |
| 2 | 120,000 | (40,000) | 80,000 | 0.7561 | 60,488 |
| 3 | 100,000 | (60,000) | 40,000 | 0.6575 | 26,300 |
| 4 | 70,000 | (40,000) | 30,000 | 0.5718 | 17,154 |
| **NPV** | | | | | **$40,902** |

**Decision:** The project should be accepted, this is because it increases Mr. Ariel wealth by $40, 902.

## ILLUSTRATION 5

Calculate the NPV for a project which has an initial outlay of $3,000 and generates annual cash inflows of $1,000, $900, $800, $700 and $600 in five years at a discount rate of 10%. Rounding up discounting factor and present value to 3 decimal places.

## SOLUTION

**How to solve**
Note this question comes with an instruction, which you have to obey "(3.d.p)".

$$NPV = \sum \frac{A_t}{(1+r)^t} - Co$$

Where
$A_t$ = cash flow per period
$t$ = period or year
$r$ = cost of capital or discount rate
$Co$ = capital outlay

| Period | Cash flows $ | D. c. f 10% | PV $ |
|--------|--------------|-------------|------|
| 0 | (3,000) | 1 | (3,000) |
| 1 | 1,000 | 0.909 | 909 |
| 2 | 900 | 0.826 | 743 |
| 3 | 800 | 0.751 | 601 |
| 4 | 700 | 0.683 | 478 |
| 5 | 600 | 0.621 | 373 |
| | | | $104 |

**Decision:** The project should be accepted, this is because it increases the shareholders' wealth by $104. Note I used 3 decimal places for my D.C.F at 10%.

## ILLUSTRATION 6

KK Ltd. is planning to invest in a project costing $30,000. The expected annual cash flows of the investment for the next 30 years is $3,000 inflow at a discount rate of 10%. Using the NPV determine the acceptability of the project.

## SOLUTION
**How to solve**

This question is just like any other NPV question except that it has a lengthy cash flow period. The question then is, are we laboriously going through this lengthy process?

| Period | Cash flows $ | D.C.F @ 10% | Present value $ |
|--------|--------------|-------------|-----------------|
| 0 | (30,000) | | |
| 1 | 3,000 | | |
| 2 | 3,000 | | |
| 3 | 3,000 | | |
| 4 | 3,000 | | |
| 5 | 3,000 | | |
| 6 | 3,000 | | |
| 7 | 3,000 | | |
| 8 | 3,000 | | |
| 9 | 3,000 | | |
| 10 | 3,000 | | |
| 11 | 3,000 | | |
| 12 | 3,000 | | |
| 13 | 3,000 | | |
| 14 | 3,000 | | |
| 15 | 3,000 | | |
| 16 | 3,000 | | |
| 17 | 3,000 | | |
| 18 | 3,000 | | |
| 19 | 3,000 | | |
| 20 | 3,000 | | |
| 21 | 3,000 | | |
| 22 | 3,000 | | |
| 23 | 3,000 | | |
| 24 | 3,000 | | |
| 25 | 3,000 | | |
| 26 | 3,000 | | |

| | | | |
|---|---|---|---|
| 27 | 3,000 | | |
| 28 | 3,000 | | |
| 29 | 3,000 | | |
| 30 | 3,000 | | |

The answer to that question is **No**. Except if told to do so.

Check this secret out. If you have questions on NPV that have exactly the same cash flows, no matter the number of years, there is a formula you can use to solve it. See below:

$$\text{NPV} = \frac{CF (1 - (1 + r)^{-n})}{r} \quad - \quad Co$$

Where $CF$ = Cash flow = $3,000
  $r$ = Cost of capital = 10% or 0.1
  $n$ = No of years = 30
  $Co$ = Capital outlay = $30,000

To solve the illustration above, we input the figures thus:

$$\text{NPV} = \frac{\$3,000 (1 - (1 + 0.1)^{-30})}{0.1} \quad - \quad \$30,000$$

NPV =N 28,280.74 – $30,000
NPV = -1,719.26

**Decision:** Therefore we reject the project.

Please note that this formula can only be applied <u>when the cash flows are exactly the same.</u>

**Method 2**: Using a shorter tabular method with cumulative distributing factor.

| Period | C.F $ | D.C.F | P.V $ |
|---|---|---|---|
| 0 | (30,000) | 1 | (30,000) |
| 1 – 30 | 3,000 | | |
| | | | |
| | | | |

For the period (1 – 30) row in the table above, use the cumulative discounting factor formula**.**

$$\text{CDCF} = \frac{(1 - (1 + r)^{-n})}{r}$$

$$\text{CDCF} = \frac{(1 - (1 + 0.1)^{-30})}{0.1}$$

$$\text{CDCF} = \frac{(1 - (1.1)^{-30})}{0.1}$$

CDCF =   9.42691446699

Now, input this value into the discounting factor column accordingly.

| Period | C.F $ | D.C.F | P.V |
|--------|-------|-------|-----|
| 0 | (30,000) | 1 | (30,000) |
| 1 – 30 | 3,000 | 9.42691446699 | 28,280.74 |
| **NPV** | | | **-1,719.26** |

So you see, the same answer.

Key Note:
If you have a question on NPV in this format, where you are told to solve with a certain discount rate. E.g 15%

| Projects | Cash flows | | | | | |
|----------|------|------|------|------|------|------|
| | Yr 0 | Yr 1 | Yr 2 | Yr 3 | Yr 4 | Yr 5 |
| A | 1,000 | 1,000 | 500 | 400 | 100 | - |
| B | 6,000 | 1,000 | 1,000 | 4,000 | 1,000 | 1,000 |
| C | 3,000 | 1,000 | 1,000 | - | 1,000 | 1,000 |

The vital issues I would like to address here are:
1   You have to calculate the NPV per project separately, using your normal 4 columnar table.
2   Do not also forget that anywhere you see year 0, that figure represents the initial outlay for that project.
3   See table below:

Project A

| Period | Cash flows $ | D.C.F | PV |
|--------|-------------|-------|-----|
| 0 | (1000) | | |
| 1 | 1,000 | | |
| 2 | 500 | | |
| 3 | 400 | | |
| 4 | 100 | | |
| 5 | - | | |

Then solve normally with the given discount rate.

Project B

| Period | Cash flows $ | D.C.F | PV |
|--------|-------------|-------|-----|
| 0 | (6000) | | |
| 1 | 1000 | | |
| 2 | 1000 | | |
| 3 | 4000 | | |

| 4 | 1000 | | |
|---|------|---|---|
| 5 | 1000 | | |

Then solve normally with the given discount rate

Project C

| Period | Cash flows $ | D.C.F | PV |
|--------|--------------|-------|-----|
| 0 | (3000) | | |
| 1 | 1000 | | |
| 2 | 1000 | | |
| 3 | - | | |
| 4 | 1000 | | |
| 5 | 1000 | | |

Then solve normally with the given discount rate.

3   After getting the individual NPVs for A, B, C, the project with the highest NPV will be selected.
4   Try it on your own let's see.

NOTE: when told to solve NPV and the question is intentionally silent about a discount rate, then you have to <u>assume 10% discount rate.</u>


## 5.14 INTERNAL RATE OF RETURN (IRR)
The internal rate of return is one controversial aspect of investment appraisal technique. This is so because of the many interesting features it possesses. First, its solution depends on two NPVs, one being positive and the other negative. Furthermore, it is a relative measure of investment appraisal that assumes discounting factors, while the NPV is absolute. At some point, instead of presenting one rate it actually give you multiple IRRs, and it is a well known arch-rival of the NPV. So you see why this guy raises so much dust? Well, let us meet the IRR.

The IRR is a discounted cash flow technique that is obtained where the net present value of all the cash flows (both positive and negative) from a project equal zero. Meaning, that discount rate, when used on a project will give us the Net Present Value (NPV) of zero (0) or the point where the present value is equal to the initial outlay.
1.   NPV = 0.
2.   PV = Co

The IRR formula in it simplest state is:

$$IRR = LDF + \left[ \frac{+NPV}{+NPV - -NPV} \right] (HDF - LDF)$$

Where:
HDF            = Higher Discounting Factor
LDF            = Lower Discounting Factor
+NPV  = Positive NPV
-NPV   = Negative NPV

Decision rule:
If calculated IRR > given cost of capital = Accept the project
If calculated IRR< given cost of capital = Reject the project
If calculated IRR = given cost of capital = Accept the project

For mutually exclusive project, accept Higher **IRR**

## 5.15  ADVANTAGES OF THE IRR
1.    It takes time value of money into account.
2.    The rates for appraising projects are given.
3.    It considers the cash flows of the entire project.

## 5.16  DISADVANTAGES OF THE IRR
1.   The ultimate or unique answer may not be derived.
2.   It is not too easy to understand.
3.   When projects have different life span, IRR may not fall in line with that of the NPV.

## ILLUSTRATION 1

Prof Edet, a lecturer in Harvard Business School is considering an investment in the oil industry costing $5,500.  The cash flows for the project are as follows:

| Period | Cash flows $ |
|--------|--------------|
| 1 | 3,000 |
| 2 | 1,500 |
| 3 | 1,000 |
| 4 | 1,200 |

Assuming a cost of capital of 20%, you are required to calculate the IRR of the project and decide its acceptability.

## SOLUTION

**How To Solve**
The IRR rule is very simple. To solve for IRR you need to get two NPVs.  **Compulsorily** one of them must be positive and the other must be negative before you can then input these values in the IRR formula.

The systematic processes for the IRR method are:
1.  Calculate the NPV normally with the cost of capital provided in the question (20%).

| Period | Cash flows $ | Discounting factor 20% | Present value $ |
|--------|--------------|------------------------|-----------------|
| 0 | (5,500) | 1 | (5,500) |
| 1 | 3,000 | 0.8333 | 2,499.9 |
| 2 | 1,500 | 0.6944 | 1,041.6 |
| 3 | 1,000 | 0.5787 | 578.7 |
| 4 | 1,200 | 0.4823 | 578.76 |
| **NPV** | | | **-801.04** |

NPV @ 20% = -5,500 + 2,499.9 + 1,041.6 + 578.7 + 578.76
            = - 801.04
NPV @ 20% = -801.04

You cannot decide now, you have to move to the next step.

2. Since we got a negative NPV initially, the next step is <u>to get a positive NPV. This can be done by reducing the cost of capital by 10%.</u> i.e. 20% - 10% = 10%, that will be used for getting a positive NPV. Note there is no hard and fast rule as to the rate, it is arbitrary or done via trial and error.

I will like to state here that:
a) If the first calculated **NPV is negative**, then you need to **reduce the cost of capital in the question by 5% or 10% or 15% etc**, the important thing to note is that we need to get the next NPV to be positive. However, most times I prefer to use 10% for large cost of capital or 5% for small cost of capital given in a question.
b) If the first calculated **NPV is positive**, then **increase the cost of capital in the question by 5% or 10% or 15% etc**, the important thing is that we need to get the next NPV to be negative.
c) as stated earlier, there is no ironclad rule as to the values to be added or deducted from the cost of capital (it is simply a trial and error method), the important thing is that the objective of positivity and negativity of NPVs must be achieved.

3. Calculate the second NPV with the new cost of capital of 10% as shown in point 2 above, to get a positive NPV.

| Period | Cash flows $ | Discounting factor 10% | Present value $ |
|---|---|---|---|
| 0 | (5,500) | 1 | (5500) |
| 1 | 3,000 | 0.9091 | 2,727.3 |
| 2 | 1,500 | 0.8264 | 1,239.6 |
| 3 | 1,000 | 0.7513 | 751.3 |
| 4 | 1,200 | 0.6830 | 819.6 |
| **NPV** | | | **37.8** |

NPV @ 10%  = -5,500 + 2,727.3 + 1,239.6 + 751.3 + 819.6
            = 37.8
NPV @ 10%  = 37.8
You would have noticed now that we have achieved the ingredients needed to solve for IRR. These are:

a.    Higher discounting factor
b.    Lower discounting factor
c.    Positive NPV
d.    Negative NPV

Therefore,          20%   =       -801.04
                    10%   =        37.8
Where HDF     = higher discounting factor is          20%

LDF   = lower discounting factor is        10%
+NPV  = positive NPV is                 37.8
-NPV  = Negative NPV is              -801.04

These will now be inputted in the IRR formula:

$$IRR = LDF + \left[ \frac{+NPV}{+NPV - -NPV} \right] (HDF - LDF)$$

$$IRR = 10\% + \left[ \frac{37.8}{37.8 - (-801.04)} \right] (20\% - 10\%)$$

$$IRR = 10\% + \left[ \frac{37.8}{838.84} \right] (10\%)$$

IRR = 10% + (0.04506)(10%)
IRR = 0.10 + (0.04506)(0.10)
IRR = 0.10 + 0.004506
IRR = 0.10456 or 0.1046 or 10.46 % or 10.5% when expressed in %.

"C'est fini" just like the French would say.

**Decision:**
Since the calculated (IRR) of 10.5% is less than the cost of capital given in the question (20%), what we need do is to reject the project. This rejection was earlier confirmed by the negative NPV result

**METHOD 2**

This second method has just been recently formulated and I am yet to present it to the **international body of finance** to approve it for global use but it helps in determining the closest discount rate that we can use to get our first NPV before the second NPV is then calculated.

Lets apply this formula towards solving our IRR by getting our first discounting factor.

$$1^{st} DF = \frac{400(\sum CF - Co + Sv)}{3n (Co + Sv)}$$

Where   DF = Discounting Factor.
         400 = Constant value.
        $\sum CF$= Addition of all the cash flows.
          Co = Capital outlay.
        Sv= Scrap value.
          n = Number of years.
          3 = Constant value.

Inputting the values we have:

$$1^{st} DF = \frac{400 ( 6,700 - 5,500 + 0)}{3x4 (5500 + 0)}$$

$1^{st}$ DF = $\dfrac{480,000}{66,000}$ = 7.27%

$1^{st}$ DF = 7.27%, rounding it off to the nearest whole number it will give 7%. This means that I should start my first NPV with a discount rate of 7%.

| Period | Cash flows $ | Discounting factor 7% | Present value $ |
|---|---|---|---|
| 0 | (5,500) | 1 | (5500) |
| 1 | 3,000 | 0.9346 | 2,803.8 |
| 2 | 1,500 | 0.8734 | 1,310.1 |
| 3 | 1,000 | 0.8163 | 816.3 |
| 4 | 1,200 | 0.7629 | 915.48 |
| **NPV** | | | **345.68** |

Since we got a positive NPV, the next step to be taken is to get a negative NPV by increasing the cost of capital by say 5%, since there is no rule as to the exact value to be added I would just use 5% i.e. 7%+ 5% = 12%, that will be used for getting a negative NPV.

Now using a discounting factor of 12%. I would solve for the negative NPV.

| Period | Cash flows $ | Discounting factor @ 12% | Present value $ |
|---|---|---|---|
| 0 | (5,500) | 1 | (5,500) |
| 1 | 3,000 | 0.8929 | 2,678.7 |
| 2 | 1,500 | 0.7972 | 1,195.8 |
| 3 | 1,000 | 0.7118 | 711.8 |
| 4 | 1,200 | 0.6355 | 762.6 |
| **NPV** | | | **-151.1** |

Using the formula:
These will now be inputted in the IRR formula:

$$\textbf{IRR} = \textbf{LDF} + \left( \frac{\textbf{+NPV}}{\textbf{+NPV - -NPV}} \right) \textbf{(HDF – LDF)}$$

$$\text{IRR} = 7\% + \left( \frac{345.68}{345.68- (-151.1)} \right) (12\% - 7\%)$$

IRR = 7% + (0.69584) (5%)
IRR = 0.07 + (0.69584) (0.05)
IRR = 0.07 + 0.0348

IRR = 0.1048 or 0.105 or 10.5% (the same with what we got earlier).

## 5.17 PROFITABILITY INDEX (PI)

The Profitability Index (PI) is a useful tool for ranking projects because it allows one to quantify the amount of value created by the project per unit of investment. It is also called the benefit cost ratio. It is calculated by dividing the present value of the project's future cash flows by the initial investment.

PI = $\underline{\text{Present Value}}$     or     $\underline{\text{PV}}$
    Capital outlay           Co

or

PI = $\underline{\text{NPV} - (- \text{Capital outlay})}$     or     $\underline{\text{NPV} + \text{Co}}$
         Capital outlay              Co

Decision Rule:
If PI > 1 accept project
If PI < 1 reject project

## ILLUSTRATION 1

Kemi Olatunji is considering a project whose capital outlay is $20,000. If the cash flows to be generated for the next 4 years are $10,000, $20,000, $15,000, and $8,000 respectively. Using a 10% cost of capital, calculate the profitability index (PI) of the project, suggesting whether the project should be accepted or rejected.

## SOLUTION

**How to solve**

| Period | Cash flows $ | Discounting factor 10% | Present value $ |
|--------|--------------|------------------------|-----------------|
| 1 | 10,000 | 0.9091 | 9,091 |
| 2 | 20,000 | 0.8264 | 16,528 |
| 3 | 15,000 | 0.7513 | 11,269.5 |
| 4 | 8,000 | 0.6830 | 5,464 |
| $\sum$PV | | | **42,352.5** |

Notice that I did not include the initial outlay in my calculation. That is one very important point I need you to note.

PI = $\underline{\text{Present Value}}$
     Capital outlay

PI = $\underline{9{,}091 + 16{,}528 + 11{,}269.5 + 5{,}464}$
              20,000

PI = $\underline{42{,}352.5}$
     20,000

PI = 2.12

**Decision:** Accept the project, because PI is > 1.

*Note: I started my table from year 1 instead of year zero (0), this was done to avoid the mistake in computation under examination condition.*

Another way PI can also be calculated is using the Net Present Value table

| Period | Cash flows $ | Discounting factor 10% | Present value $ |
|--------|--------------|------------------------|-----------------|
| 0 | (20,000) | 1 | (20,000) |
| 1 | 10,000 | 0.9091 | 9,091 |
| 2 | 20,000 | 0.8264 | 16,528 |
| 3 | 15,000 | 0.7513 | 11,269.5 |
| 4 | 8,000 | 0.6830 | 5,464 |
| **NPV** | | | **22,352.5** |

PI = $\frac{\text{NPV} - (-\text{ Capital outlay})}{\text{Capital outlay}}$    or    $\frac{\text{NPV} + \text{Co}}{\text{Co}}$

This formula is also useful when (NPV and Co) only are given without cash flows, as would be seen in Illustration 2 below.

PI = $\frac{(22,352.5 - - 20,000)}{20000}$

PI = $\frac{(22,352.5 + 20,000)}{20000}$

PI = $\frac{42,352.5}{20,000}$

PI = 2.12

**ILLUSTRATION 2**

| PROJECTS | A $ | B $ |
|----------|-----|-----|
| COST | (40,000) | (30,000) |
| NPV | 50,000 | 59,000 |

Use the PI to determine the project to be selected based on ranking

**SOLUTION**

**How to solve**

Project A
$$PI = \frac{NPV - (- \text{Capital outlay})}{\text{Capital outlay}} \quad \text{or} \quad \frac{NPV + Co}{Co}$$

$$PI = \frac{50{,}000 - (- 40{,}000)}{40{,}000}$$

$$PI = \frac{50{,}000 + 40{,}000}{40{,}000}$$

$$PI = \frac{90{,}000}{40{,}000}$$

$$PI = 2.25$$

PI of Project A = 2.25

Project B
$$PI = \frac{NPV - (- \text{Capital outlay})}{\text{Capital outlay}} \quad \text{or} \quad \frac{NPV + Co}{Co}$$

$$PI = \frac{59000 - (- 30000)}{30000}$$

$$PI = \frac{89000}{30000}$$

$$PI = 2.97$$

PI of Project B = 2.97

Project B should be selected because it has a higher profitability index

Note: Ignore the negative sign (-) for the capital outlay in the denominator of the formula.

**Note**: If the cash flows provided in the question are the same all through the year you can use the formula below to solve for your PI. It is also helpful for the multiple choice questions, as used in the multiple choices section after this topic in this book .

$$PI = \frac{CF\,(1 - (1 + r)^{-n})}{r\,(Co)}$$

Where CF =  One cash flows out of the constant cash flow.
        r  =  Cost of capital.

n  = Number of year.
Co = Capital outlay.

Note: This formula should only be used when you have a constant cash flows and for multiple choices only.

## 5.18 ACCOUNTING RATE OF RETURN (ARR)

The Accounting Rate of Return (ARR) is the ratio of profit before interest and tax to percentage of capital employed at the end of a period. The ARR provides a quick estimate of a project's worth over its useful life. It is used by management to determine cash inflow on a business venture or project. It is useful for individual project's profitability.

## 5.19 MERITS OF ARR

1. It is simple to calculate.
2. It is easy to understand.
3. It considers the total profit of a project throughout its operating life.
4. It expresses relations between investments and  in terms of percentage.

## 5.20 DEMERITS OF ARR

1. it does not assume reinvestment of profits.
2. it ignores the time value of money.
3. it uses account profit and not cash flows.

The core formula for ARR is:

$$ARR = \frac{\text{Average Annual Profit}}{\text{Average Investment}} \times 100$$

Or

$$ARR = \frac{AAP}{AI} \times 100$$

Note: the AAP above will be calculated separately.
        The AI will also be calculated separately.

Decision Rule:
If ARR>RR ( Rate of Return) =  Accept Project.
If ARR<RR ( Rate of Return) =  Reject Project.

Where:
ARR   = Average Rate of Return
RR     =    Rate of Return

For mutually exclusive projects accept Higher ARR.

$$ARR = \frac{\text{Average Annual Profit}}{\text{Average Investment}} \times 100$$

Average Annual Profit (AAP) =

$$= \frac{[\text{Total profit} - (\text{cost} - \text{scrap value})]\,(1-t)}{\text{No of year}}$$

(Depreciation)

Average Investment (AI) $= \dfrac{\text{Cost of Investment} + \text{Scrap Value}}{2}$

## ILLUSTRATION 1

Mr. Zion Blake has the following company data.

| | |
|---|---|
| Cost of investment | $ 170,000 |
| Life cycle of project | 4 years |
| Scrap value | - |

| Profit during the year | $ |
|---|---|
| Yr 1 | 50,000 |
| Yr 2 | 95,000 |
| Yr 3 | 120,000 |
| Yr 4 | 105,000 |

If the company's projected rate of return is 45% should the project be accepted using the ARR?

## SOLUTION

### How to solve ARR

You have to check if the question has a tax rate, if not, then repeat your formula:

ARR = $\dfrac{\text{Average Annual Profit (AAP)}}{\text{Average Investment (AI)}}$ x 100

Or

ARR = $\dfrac{\text{AAP}}{\text{AI}}$ x 100

Total profit = 370,000
Cost = 170,000
Scrap value = 0
tax = 0
n = 4

| Step 1. Calculate Average Annual Profit (AAP) | Step 2. Calculate (Average Investment) |
|---|---|
| Depreciation<br>AAP = [Total profit – ( cost – scrap value)] (1-t)<br>No of year<br><br>AAP = [$370,000 – ( $170,000 – 0)] (1-0)<br>4<br><br>AAP = $200,000<br>4<br><br>**AAP = $50,000** | AI = Cost of Investment + Scrap Value<br>2<br><br>AI = $170,000 + 0<br>2<br><br>**AI = $85,000** |

Where AAP calculated is $50, 000
　　　AI calculated is $85, 000

ARR = $\frac{AAP}{AI}$ x 100

Therefore ARR = $\frac{\$50,000}{\$85,000}$ x 100

　　　ARR = 58.82%

**Decision:**
Since the ARR calculated is greater than the rate of return given in the question, then the project should be accepted.

**METHOD 2**

This second method has just been recently formulated, and I am yet to present it to the **International Body of Finance** to approve it for global use. It gives the same answer as the conventional method and its edge is that it is shorter and particularly helpful for objectives or multiple-choice questions.

This formula is when tax is not given.

ARR = $\frac{200(\sum P - Co + Sv)}{n (Co + Sv)}$

Where 200　　= Constant Value
　　　$\sum P$　　= Addition of all the profits
　　　Co　　= Capital outlay
　　　Sv　　= Scrap value
　　　n　　= number of years

Inputting values from the first Illustration above we have:

$$ARR = \frac{200(\ 370{,}000 - 170{,}000 + 0)}{4(170{,}000 + 0)}$$

ARR = 58.82%

The beautiful thing about this formula is that it is faster and gives your answer straight up in percentage.

## ILLUSTRATION 2

Stone and Cousin are partners in a transport business. The following data relates to their company's investment:

| Year | Sales $ | Overhead $ |
|---|---|---|
| 1 | 5,000 | 2,000 |
| 2 | 10,000 | 1,000 |
| 3 | 15,000 | 500 |

The cost of the business is estimated at $28,000, and a scrap value of the equipment is $2,100.

Using the ARR, what should be the company's decision if the profit rate of return expected is 10%?

## SOLUTION

**How to solve**

Taking a close look at this question you will notice that a major requirement needed for our computation is not given i.e. profit. Therefore, we have to calculate for that first, by deducting the overhead from the sales per period.

**Calculate for profit**

| Year | Sales $ - Overhead $ | Net profit $ |
|---|---|---|
| **1** | 5,000 - 2,000 | 3,000 |
| **2** | 10,000 - 1,000 | 9,000 |
| **3** | 15,000 - 500 | 14,500 |
| **Total net profit** | | **26,500** |

| Year | Sales $ | | Overhead $ | Net profit $ |
|---|---|---|---|---|
| 1 | 5000 | - | 2000 | 3000 |
| 2 | 10000 | - | 1000 | 9000 |
| 3 | 15000 | - | 500 | 14500 |
| **Total net profit** | | | | **26500** |

With this net profit, we can now start solving for our ARR.

How to solve ARR.
You need to check if the question has a tax rate, in this case it does not.

$$ARR = \frac{\text{Average Annual Profit (AAP)}}{\text{Average Investment (AI)}} \times 100$$

Cost = $28,000
Total profit = $26,500
Scrap value = $2,100
Tax = 0
n = 3

| Step 1. Calculate Average Annual Profit (AAP) | Step 2. Calculate (Average Investment) |
|---|---|
| Depreciation<br>AAP = $\frac{[\text{Total profit} - (\text{cost} - \text{scrap value})](1-t)}{\text{No of year}}$<br><br>AAP = $\frac{[26,500 - (28,000 - 2,100)](1-0)}{3}$<br><br>AAP = $\frac{600}{3}$<br><br>**AAP = $200** | $AI = \frac{\text{Cost of Investment} + \text{Scrap Value}}{2}$<br><br>$AI = \frac{28,000 + 2,100}{2}$<br><br>$AI = \frac{\$30,100}{2}$<br><br>**AI = $15,050** |

Note:
- I used 0 as value for t (tax), since it wasn't given.
- I used $2,100 as value for scrap.

Step 3
We then use the values of AAP and AI we got above to solve for our ARR.

$$ARR = \frac{AAP}{AI} \times 100$$

Where AAP calculated is $200.
     AI calculated is $15,050,

$$ARR = \frac{200}{15,050} \times 100$$

ARR = 1.33%

**Decision:**
Since the ARR calculated is less than the rate of return, then the project should be rejected.

**Method 2 (Formula Method)**

$$ARR = \frac{200(\sum P - Co + Sv)}{n\,(Co + Sv)}$$

Where        200     = Constant Value
             $\sum P$ = Addition of all the profits
             Co      = Capital outlay
             Sv      = Scrap value
             N       = number of years

Using the values for illustration 2

$$ARR = \frac{200\,(26{,}500 - 28{,}000 + 2{,}100)}{3\,(28{,}000 + 2{,}100)}$$

$$ARR = \frac{120{,}000}{90{,}300}$$

ARR = 1.33%

## 5.21   ARR WHEN TAX IS GIVEN

$$ARR = \frac{\text{Average Annual Profit}}{\text{Average Investment}} \times 100$$

$$\text{Average Annual Profit} = \frac{[(\text{Total profit} - (\text{cost} - \text{scrap})](1\text{-}t)}{\text{No of years}}$$

$$\text{Average Investment} = \frac{\text{Cost of Investment} + \text{Scrap value}}{2}$$

## ILLUSTRATION 3

Ben Johnson has invested $50,000 in a project, which has the following analysis.
Life cycle of project          5 years
Residual value                 3,700
Tax rate                       10%

| Profit during the year | $ |
|---|---|
| Year 1 | 10,000 |
| Year 2 | 15,000 |
| Year 3 | 20,000 |
| Year 4 | 15,000 |
| Year 5 | 10,000 |

If the company projected rate of return is 12%. Should the project be accepted using ARR?

## SOLUTION

**How to solve ARR**

Check if the question has a tax rate, in this case there is a tax rate of 10%.

ARR =  Average Annual Profit (AAP)    x    100
          Average Investment (AI)

| Step 1. Calculate Average Annual Profit (AAP) | Step 2. Calculate (Average Investment) |
|---|---|
| Depreciation<br>AAP = [Total profit – ( cost – scrap value)] (1-t)<br>No of year<br><br>AAP = [$70,000 – ( $50,000 – $3,700)] (1-0.1)<br>5<br><br>AAP =   $21,330<br>5<br><br>**AAP = $4,266** | AI  =  Cost of Investment + Scrap Value<br>2<br><br>AI  =  $50,000 +  $3,700<br>2<br><br>AI  = $53,700<br>2<br><br>**AI = $26,850** |

Note:
- I used  0.10  as value for t (tax), since it was  given as 10%.
- I used  $3,700  as value for scrap, since it was  given.
- Be mindful of the "double brackets" for the AAP above, you will get it wrong if you ignore them.  This is so because, of the application of tax on the total amount.

**Step 3**

Finally to get ARR =    AAP            X       100
                                   AI

ARR =   $4,266   x  100
           $26,850

ARR = 15.9%

**Decision:** The ARR calculated is greater than the expected return we therefore ACCEPT the project.

## METHOD 2

This second method has just been recently formulated, and I am yet to present it to the **International Body of Finance** to approve it for global use but it gives the same answer as the conventional method. Its edge is that it is fast and particularly helpful for objective or multiple-choice questions.

$$ARR = \frac{200(\sum P - Co + Sv)(1 - t)}{n\,(Co + Sv)}$$

Where 200     = Constant
       $\sum P$    = Addition of all the profits
       Co    = Capital outlay
       Sv    = Scarp value
       t    = tax rate
       n    = number of years

Inputting the values of the immediate question we now have,

$$ARR = \frac{200\,(\$70{,}000 - \$50{,}000 + \$3{,}700)\,(1 - 0.10)}{5(\$50{,}000 + \$3{,}700)}$$

$$ARR = \frac{\$4{,}266{,}000}{\$268{,}500}$$

ARR = 15.9%